How to get more pupils from level 3 to level 5 in mathematics – part 2

Course handbook
Acknowledgements

We are grateful to:

- Malcolm Swan for his work on learning from misconceptions in mathematics;
- the North East consultants’ group led by Davina Jones of Sunderland LEA for the 10-4-10 materials;
- Geoff Fowler of Birmingham LEA for the work done in producing the TS materials.

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The websites referred to in these materials existed at the time of going to print. Teachers should check all website references carefully to see if they have changed and substitute other references where appropriate.
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Introduction

The training consists of three sessions.

• Session 1 is based around mistakes that pupils make in mathematics. Sometimes these mistakes are careless errors but often they are the result of misconceptions. Common misconceptions are examined and teaching strategies to address and prevent them are considered. Materials are provided to support course participants in running a similar session back in school.

• Session 2 introduces the booklet *Teaching mental calculation strategies to level 5* (DfES 0744-2004). Many pupils who enter Key Stage 3 at level 3 in mathematics do not have the skills of mental calculation to make real progress. The booklet systematically introduces teachers to these skills and provides teaching activities to reinforce them.

• Session 3 builds on *Securing level 5 in mathematics: Year 9 intervention* (DfES 0293-2004). Additional materials are provided to support test preparation for Year 9 pupils.

The sessions aim to:

• support schools in using pupils’ misconceptions to develop teaching and learning;

• support the teaching of mental strategies to help ensure that pupils attain at least in line with expectations;

• build on materials to support pupils:
  – *Targeting level 4 in Year 7: mathematics* (DFE 0085-2003);
  – *From level 4 to level 5 in mathematics: Year 8 intervention* (DFE 0292-2004);
  – *Securing level 5 in mathematics: Year 9 intervention* (DFE 0293-2004);

• support schools in incorporating the activities and approaches into their own teaching programmes.

After the course, you will be expected to:

• ensure that all pupils make progress during Key Stage 3:
  – check that measures are in place to tackle underachievement in Years 7, 8 and 9;
  – use the Key Stage 3 Strategy materials;

• ensure that teachers and learning support assistants are introduced to the new materials and plan how the materials and training will be used to improve practice in the department;

• consider running a session on errors and misconceptions in school;

• discuss the use of the booklet *Teaching mental calculation strategies to level 5*, and if necessary order further copies from QCA (for ordering details contact QCA Enterprises on 0207 509 5389 or visit the QCA website at www.qca.org.uk/products).
These resources are intended to support departmental meetings in identifying and developing approaches to tackle pupils’ misconceptions in mathematics. Four topics are included on the CD-ROM:

A Fractions and decimals;
B Multiplication and division;
C Area and perimeter;
D Algebraic notation.

Each topic includes:
1 examples of errors from a pupil’s work;
2 a summary by researchers of the misconceptions evident in the pupil’s work;
3 Key Stage 3 test questions relating to the specified topic;
4 an outline of an approach to uncover misconceptions;
5 a card sort activity to enable pupils to discuss their different interpretations.

The activities associated with this approach involve pupils in groups discussing their solutions. Two types of activity are illustrated in these materials:

- collecting together different but equivalent representations of a concept or process (e.g. activities in topics A and B);
- testing the validity of generalisations by asking whether they are always, sometimes or never true (e.g. activities in topics C and D).

The discussion and reflection generated by both types of activity contribute to pupils’ learning as they share, exchange and clarify meanings and interpretations.

The examples are illustrations only. If you have already discussed your pupils’ misconceptions, perhaps through analysis of test scripts, use those samples to consider changes in approach that would most benefit your pupils.
How to use these resources

In a departmental meeting:
• consider the pupil’s responses to the questions set (resource D1) and discuss the misconceptions that are evident;
• compare your response with the findings from the researchers (resource D2);
• predict how the pupil might answer an associated test question (resource D3);
• explore approaches that target the misconceptions (resource D4) and do the card sort activity (resource D5);
• discuss likely outcomes from pupils’ discussions when they use the card sort;
• consider how to use pupils’ responses to create and resolve the cognitive conflict by encouraging them to discuss their imagery and reasoning.

In teaching:
• consolidate skills by setting questions that focus on the application of the newly acquired concept;
• adjust your schemes of work to incorporate the activities designed to counter misconceptions.
TOPIC D  
Algebraic notation

Samples of a pupil’s work

6. A piece of rope 5 metres long is cut into two pieces. 
One piece is $x$ metres long. 
How long is the other piece? 

7.5 metres.................................

There are 24 hours in one day. 
How many hours are there in $y$ days? 

$y \times 24 = 72$ hours......................

It costs £140 to hire a coach. 
This cost is shared equally among $n$ people. 
How much does each person pay? 

$n \times 140 = £10$ each......................

A plumber charges £30 to come to your house plus an extra £20 for each hour that the job takes. 
A job takes $x$ hours. 
How much does the plumber charge? 

$x \times 3 = £90$..............................

7. What can you say about $x$ if $x + y = 10$ and $x$ is less than $y$?

$x = 4$.........$y = 6$.........$4 + 6 = 10$.

9. A piece of rope 60 metres long is cut into two pieces. 
One piece is $x$ metres long and the other is $y$ metres long.

Write down two equations. 
Each equation should use $x$, $y$ and 60.

$x + y = 60$

$x = 30$..............................

13. If $y = 1 + 4x$ and $x = 3$
then $y = 1 + 4 \times 3 = 13$ ..............................

If $A = 3x^2$ and $r = 4$
then $A = 3 \times 4 \times 4 = 48$ ..............................
Commentary on the pupil’s work

The pupil’s answers to questions 6, 7 and 9 show that she does not recognise that letters represent variables. In every case, she substitutes particular values for the letter. In questions 6, 7 and 9, she substitutes 3, 4 and 30 for \( x \), so there is some realisation that \( x \) can take different values in different questions, but she does not allow for this within a single question.

Notice also how she has let \( n = 14 \) in question 6. This is presumably because \( n \) is the 14th letter in the alphabet. This is reminiscent of children’s secret codes, where \( a = 1 \), \( b = 2 \) and so on.

Her responses reveal a general reluctance to leave operations in answers. She appears to think that if an operation is present then something still needs to be done.

Her answer to question 13 shows that she does not recognise the conventions of algebra: that multiplication precedes addition and that squaring should precede multiplication.

Her use of the equals sign is idiosyncratic. As with many pupils, she writes such things as \( 1 + 4 = 5 \times 3 = 15 \) while evaluating an expression. This tendency is consistent with an interpretation of the ‘\( = \)’ symbol as meaning ‘makes’ – a signal to evaluate what has gone before. This is the same meaning as the button with this label on a calculator.
Key Stage 3 test questions

1 Simplify these expressions.

\[5k + 7 + 3k = \quad \]

\[k + 1 + k + 4 = \quad \]

2 Solve these equations.

\[8k - 1 = 15 \quad k = \quad \]

\[2m + 5 = 10 \quad m = \quad \]

3 You can work out the cost of an advert in a newspaper by using this formula:

\[C = 15n + 75\]

\(C\) is the cost in pounds.

\(n\) is the number of words in the advert.

(a) An advert has 18 words.

Work out the cost of the advert.

Show your working.

£

(b) The cost of an advert is £615.

How many words are in the advert?

Show your working.
Always, sometimes or never true?

Answer these questions on your own, without talking to your neighbour. If you get stuck, don’t worry. You can come back to them later. In each question, \( n \) stands for any number.

1. When are the following true – always, never or sometimes? If you choose ‘sometimes true’ then say when it is true.

<table>
<thead>
<tr>
<th>Always true</th>
<th>Sometimes true when …</th>
<th>Never true</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n + 2 = 3 )</td>
<td>No</td>
<td>Yes, when ( n = 1 )</td>
</tr>
<tr>
<td>( 2n - 3 = 3 - 2n )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( n + 12 = m + 12 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 3(n + 3) = 3n + 3 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( n^2 ) is greater than ( n )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 4n ) is greater than ( 4 + n )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Now try the following activity in a small group. You will need the cards for sorting from resource D5.

   Decide whether each of the statements on the cards is
   - Always true (it is true for all possible values)
   - Sometimes true (it is true for just some values)
   - Never true (no values make the statement true)

   Take it in turns in your group to explain your thinking.
   Explain using examples how you made your decision.
   If you think a statement is sometimes true, say which numbers make it true. If you think a statement is never true, then say how you can be sure.
   Statements which are always true are called identities.
   When you reach agreement, stick the statement into the correct column.

On your own, go back and revise your answers to question 1.
Make notes on any mistakes you made and the reasons for them.
Make notes on new things you have learned.
## Equations for sorting

<table>
<thead>
<tr>
<th>D1</th>
<th>D2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a + 5 = 12$</td>
<td>$b + 12 = b + 16$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D3</th>
<th>D4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2c + 3 = 3 + 2c$</td>
<td>$2d - 5 = 5 - 2d$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D5</th>
<th>D6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4 + 2e = 6e$</td>
<td>$f + 12 = g + 12$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D7</th>
<th>D8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4h &gt; 4 + h$</td>
<td>$k + 5 &lt; 20$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D9</th>
<th>D10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3(m + 3) = 3m + 3$</td>
<td>$4(3 + n) = 12 + 4n$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D11</th>
<th>D12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p^2 &gt; 4$</td>
<td>$q^2 = 10q$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D13</th>
<th>D14</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r^2 &gt; r$</td>
<td>$16s^2 = (4s)^2$</td>
</tr>
</tbody>
</table>
• Lessons should focus on known, specific difficulties. Rather than posing many questions in one session, it is better to focus on one problem or context and encourage a variety of interpretations to emerge, so that pupils can compare and evaluate them.

• Questions or stimuli should be posed in ways that create a tension or cognitive conflict that needs resolving. Contradictions arising from conflicting methods or opinions create an awareness that something needs to be learned.

• Activities should provide opportunities for meaningful feedback to the pupil on his or her interpretations. This usually involves some form of small-group discussion.

• Lessons should include time for whole-class discussion in which new ideas and concepts are allowed to emerge. This can be a complex business and requires sensitivity on the part of the teacher so that pupils are encouraged to share tentative ideas in a non-threatening environment.

• Opportunities should be provided for pupils to consolidate what has been learned through the application of the newly constructed concept.

Further reading
Ryan, J. and Williams, J. (2000) *Mathematical discussions with children*, University of Manchester


Source: *Learning from mistakes, misunderstandings and misconceptions in mathematics* (DfES 0527/2002)
Number and algebra

To help pupils to achieve level 5 you need to:
- consolidate their understanding of place value to eradicate mistakes such as $0.5 - 0.25 = 0.2$;
- make sure that they are confident in using efficient written methods of calculation for all four operations with whole numbers and simple examples involving decimals;
- develop appropriate methods – mental, written or calculator – to solve problems involving percentages such as:
  \[
  8\% \text{ of } £26.50 \\
  12.5\% \text{ of } £98
  \]
- give them experience and practice at using negative quantities;
- help pupils understand algebra as generalised arithmetic by developing number sentences to include, for example:
  \[3x + 2y = 7, \text{ so } 6x + 4y = ?\]
  \[x - 1 = 45, \text{ so } x + 2 = ?\]
  and understand that algebraic operations follow the same conventions and order as arithmetic operations;
- help pupils understand and simplify algebraic expressions such as:
  \[7 + 2t + 3t \text{ and } b + 7 + 2b + 10\]
- build on the use of inverse operations to find missing numbers and to solve equations either mentally or by formal methods, for example:
  \[26 - 2n = 8\]
- give practice at substitution into simple expressions, for example:
  If \(x = 1\), \(2x + 1 = ?\)
  \(m = 6\) and \(h = 7m / 2\), so \(h = ?\)
- encourage them to express generalisations algebraically and to explain generalisations by relating the terms of a number sequence back to the spatial pattern that it describes.

Pupils targeting level 6 need to:
- achieve confidence in calculating with fractions, decimals and percentages including calculating one number as a percentage of another;
- understand proportional reasoning and use it to solve problems, always being clear as to which amount represents one whole or 100%;
- understand and simplify algebraic expressions, for example:
  \[w^2 \times w \quad w^2 + 3w^2 \quad 3m - (m)\]
  Factorise \(7y + 14\)
- handle index notation, substitution and algebraic manipulation and be clear on the difference between equations and expressions – you need to explain and demonstrate these differences.
Shape, space and measures

To help pupils to achieve level 5 you need to:

- give them experiences to develop ideas of rotation, including locating the centre of rotation;
- give opportunities to develop their understanding of perimeter;
- teach the appropriate use of a calculator to solve problems involving measures, including time;
- teach the names and properties of polygons, including special quadrilaterals such as trapezium and kite;
- demonstrate that marking on diagrams any lengths or angles that are known often helps to see how to obtain the final answer;
- provide opportunities for them to explain their reasoning.

Pupils targeting level 6 need to:

- be able to solve angle problems involving properties of lines and polygons using a series of linked calculations and give reasons to justify their results;
- practise operating mentally on shapes, for example, by visualising views of 3-D shapes represented in diagrams;
- understand and use the formulae for the area and circumference of a circle, in particular that they calculate \( \pi r^2 \) and not \( (\pi r)^2 \) for the area of a circle.

Handling data and probability

You need to make sure that there is a consistent approach to handling data and graphs across all subjects in the school.

To help pupils to achieve level 5 you need to:

- use class or group discussion of a range of graphs focusing on interpreting the data presented;
- develop the use of number lines to assist in the interpretation of scales;
- explain and give practice in calculating and using appropriately the mean, median, mode and range for discrete data;
- provide opportunities to compare information and draw conclusions from two graphs and to use an average and the range to compare two distributions;
- encourage them to give the probability of an event as a fraction in its simplest form and link this fraction to a percentage.

Pupils targeting level 6 need to:

- interpret a range of statistical diagrams, including pie charts, time series and scatter diagrams, and draw inferences from them;
- identify and represent outcomes from a combination of events or experiments;
- know and use the fact that the sum of probabilities of all mutually exclusive outcomes is 1.
Loop card games involve pupils being dealt a set of cards. Each card has a question on it and an answer to a different question.

To begin the loop, a pupil reads out loud the question on their card and places it on the table. The pupil with the card that shows the corresponding correct answer reads out the answer. This pupil then reads out the question on their card for another pupil to answer, and places the card alongside the first card so that questions and answers match.

Loop cards are constructed to ensure that all questions and answers are used and the sequence arrives back at the first question. A loop will be formed after all the questions are answered.

The sets of loop cards provided in the school pack and on the CD-ROM are designed to be used with groups of pupils rather than a whole class. There are 10 cards in each set so pupils could work in groups of 10. However, it may be more effective for pupils to work in smaller groups and so have more than one card. This would be an ideal activity for a small group of pupils working with a teaching assistant.

A template is provided on the CD-ROM for teachers to make their own sets.

Asking pupils to make up a set of cards to illustrate a particular strategy is a good way of checking their understanding of a method.
Teaching mental calculation strategies

Contents

Each set of loop cards in the school pack reinforces a mental strategy in the booklet *Teaching mental calculation strategies to level 5*. Page references in brackets refer to the booklet.

Set 1 Partitioning 2: Bridging through multiples of 10 (page 22)
For example, 16 + 9 = 16 + 4 + 5, 73 – 7 = 73 – 3 – 4

Set 2 Partitioning 2: Building up to a whole number or to a tenth (page 22)
For example, 2.6 + 0.7 = 2.6 + 0.4 + 0.3

Set 3 Partitioning 3: Compensating (page 24)
Addition or subtraction of numbers that are close to a multiple of 10
For example, 34 + 9 = 34 + 10 – 1, 405 – 399 = 405 – 400 + 1

Set 4 Partitioning 3: Compensating (decimals) (page 24)
Addition or subtraction of numbers that are close to whole numbers or a whole number of tenths
For example, 5.7 + 3.9 = 5.7 + 4.0 – 0.1

Set 5 Partitioning 4: Using near doubles (page 26)
For example, 37 + 35 is double 35 and add 2

Set 6 Partitioning 4: Using near doubles (page 26)
For example, 4.7 + 4.8 is double 4.8 and subtract 0.1 or double 4.7 and add 0.1

Set 7 Partitioning 5: Bridging through numbers other than 10 (based on time) (page 28)
For example, the time 36 minutes after 2:45 can be calculated as:
2:45 and 15 minutes takes us to 3:00, then add 21 minutes to get 3:21

Set 8 Knowing multiplication and division facts to 10 × 10
(2, 3, 4, 5, 10 times tables) (page 32)

Set 9 Knowing multiplication and division facts to 10 × 10
(5, 6, 7, 8, 9 times tables) (page 32)

Set 10 Multiplying and dividing by multiples of 10 (page 36)
For example, how many seconds are there in 8 minutes?

Set 11 Doubling and halving (page 40)
For example, 14 × 20 = 14 × 10 × 2

Set 12 Fractions, decimals and percentages (page 42)
Using related facts for fractions
For example, \( \frac{1}{2} \) of 20 is 4, so \( \frac{3}{5} \) is 3 × 4

Set 13 Fractions, decimals and percentages (page 42)
For example, know that 25% = 0.25 = \( \frac{1}{4} \), find \( \frac{1}{4} \) of 36

Set 14 Fractions, decimals and percentages (pages 42–43)
Using 10% of an amount to calculate other percentages of that amount
For example 10% of 60 is 6, 70% of 60 is 7 × 6

Template for loop cards (CD-ROM only)
The 10-4-10 (‘Ten-for-ten’) materials were developed by Key Stage 3 consultants in the North East. The materials support pupils and parents/carers over the holiday period prior to the Year 9 tests. The resources have been used by LEAs in that area from 2003 and can be used in a range of ways.

The following pages show samples of the materials.

- **Materials for pupils**
  The first three pages show day 1 from the ten days’ work for ten minutes a day (hence ‘ten for ten’). They use selected test questions: five mental mathematics questions, and two longer questions. The materials for the other days are similar.

- **Materials for parents/carers**
  Next is the letter to parents/carers describing how the materials can be used for mathematics revision, and the reply slip at the bottom of the page. On the following three pages the material for parents/carers reproduces the day 1 questions but annotated with answers, guidance and hints.

The original purpose was for the pack to be used over the Easter holiday by pupils with support at home. With a little editing, the materials could be used as booster work or for after-school groups or breakfast clubs.
Mental questions

1  Multiply seven by seven.

2  How many nines are there in fifty-four?

3  What number should you add to negative three to get the answer five?

4  Add two point five to three quarters.

5  I think of a number. I call it $n$.
   I square my number and then add four.
   Write an expression to show the result.
Car parking

A car park shows this sign.

Car parking

70p

Pay using any of these coins:

10p 20p 50p

No change given

Complete the table to show all the different ways of paying exactly 70p.

<table>
<thead>
<tr>
<th>Number of 10p coins</th>
<th>Number of 20p coins</th>
<th>Number of 50p coins</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

2 marks
Numbers

Look at these number cards.

$+3 \quad 0 \quad -5 \quad +9 \quad +2 \quad -8 \quad +7 \quad -2$

(a) Choose a card to give the answer 4.

$+2 + -5 + \square = 4$

1 mark

(b) Choose a card to give the lowest possible answer.

Fill in the card below and work out the answer.

$-2 + \square = \_\_\_\_\_\_$

2 marks
Dear Parent/Carer/Friend

In order that your child has the best possible chance of gaining a level 5 in the Key Stage 3 mathematics test after the holidays, we are hoping you will encourage them to do some revision during the holiday break.

Your child has been given a booklet called ‘10-4-10’. This asks them to spend 10 minutes a day for 10 days on mathematics revision. They get the weekends off!

Every day there are five mental mathematics questions and two longer questions to complete. Pupils write their answers on the sheets, but may need some paper for extra working out.

In order to help you support them, we have produced an answer booklet. In this you will find the answers and also some hints on how to complete the questions.

Thank you for your support.

Please complete this form and return it to the school after the holiday.

Name of pupil ________________________________

School ________________________________

The pupil found the revision booklet a helpful way to revise.

Yes ☐ No ☐

I found the answer booklet helpful in supporting the pupil.

Yes ☐ No ☐

If this scheme is repeated I suggest the following changes.

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

Signed ________________________________
Mental questions

1. Multiply seven by seven.
   
   \[49\]

2. How many nines are there in fifty-four?
   
   \[54 \div 9 = 6\]

3. What number should you add to negative three to get the answer five?

   \[\begin{array}{c}
   -3 \\
   0 \\
   5 \\
   \end{array}\]

   \[8\]

4. Add two point five to three quarters.

   Either: \(2.5 = 2\frac{1}{2}\), giving \(2\frac{1}{2} + \frac{3}{4} = \frac{3}{4}\)
   
   or: \(\frac{3}{4} = 0.75\), giving \(2.5 + 0.75 = 3.25\)

   \[3\frac{1}{4} \text{ or } 3.25\]

5. I think of a number. I call it \(n\).
   I square my number and then add four.
   Write an expression to show the result.

   \[n \times n + 4 \text{ or } n^2 + 4\]

   \[n^2 + 4\]
Car parking

A car park shows this sign.

Car parking

70p

Pay using any of these coins:

10p  20p  50p

No change given

Complete the table to show all the different ways of paying exactly 70p.

<table>
<thead>
<tr>
<th>Number of 10p coins</th>
<th>Number of 20p coins</th>
<th>Number of 50p coins</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Be systematic.
Check that there are no repeats.
For example 10p + 50p + 10p
and
50p + 10p + 10p are repeats.
Numbers

Look at these number cards.

(a) Choose a card to give the answer 4.

\[ +2 + (-5) + 7 = 4 \]

1 mark

(b) Choose a card to give the lowest possible answer.

Fill in the card below and work out the answer.

\[ -2 + -8 = -10 \]

2 marks

You could use a number line to answer this question.

When adding a negative number, go to the left on the number line.
Snappers, part of the T5 (‘Targeting level 5’) materials, are short, whole-class interactive activities based around key areas of mathematics pitched at levels 4 and 5.

Snappers provide starter activities for lessons. They can be run directly through a computer or copied onto an OHT. Alternatively, in many cases, the idea can be quickly drawn onto a board. The teacher support material gives ideas for the use and development in the classroom of the OHTs provided.

1. Hundreds and thousands
2. Sports results
3. Stepping stones to percentages
4. Stepping stones to fractions
5. Year 9 maths
6. Arrows
7. Twelve days of Christmas
8. Substitution spider
9. Halving rectangles
10. Nets of cuboids
11. Angles and transformations
12. Transformations
13. Potato bar chart
14. Potato pie chart
15. Fairground games
16. Mean maths

Place value
Place value
Percentages
Fractions
Ratio
Sequences
Expressions and equations
Substitution
Area and perimeter
Nets and solids
Angles and symmetry
Transformations
Interpreting data
Pie charts
Probability
Averages

Teaching notes accompany the Snappers (see the CD-ROM), with learning objectives, suggested questions and suggestions for development.

The T5 plan (section 3.6) suggests a programme for the use of Snappers combined with the other resources, but this is only a suggestion and these activities can be used to enhance all mathematics teaching in Year 7, Year 8, Year 9 and beyond.
SNAPPERS

Place value

SNAPPER 1

Hundreds and thousands (place value)

Objectives
• Multiply and divide by 10, 100 and 1000
• Understand the impact of multiplying or dividing a number by 10, 100 and 1000

Suggested questions
1. What will happen to the answer 4032 if we use:
   - 5.6 rather than 56
   - both 5.6 and 7.2
   - 560 (10 times bigger) and 7.2 (10 times smaller)?
   and so on...
2. What happens to the unit value of each digit in 4032 when you decide to make it 10 times bigger? 100 times smaller?
3. Is it really the decimal point or the digits moving?

Suggestions for development
1. Link to problems with metric units.
2. Extend to factors other than 10, 100 and 1000. For example:
   If \(56 \times 72 = 4032\), what is \(56 \times 36\)?
3. Keep the final answer and explore relationships. For example:
   If \(56 \times 72 = 4032\), then \(28 \times ? = 4032\)

SNAPPER 2

Sports results (place value)

Objectives
• Order decimal numbers to at least two decimal places
• Add and subtract decimal numbers with at least two decimal places
• Multiply and divide by 10, 100 and 1000; convert between metric units of length

Suggested questions
1. Can you complete the sports results?
   Javelin:
   - Which digit do you look at first?
   - If that’s the same, what next?
   - Why is it tempting to say B won?
   Long jump and pole vaulting:
   - Why is it difficult to compare 5.90 m and 5.095 m?
   - Which unit shall we agree on?
2. What is the total distance thrown by all five javelin competitors?
3. By how much did the javelin thrower win?

Suggestions for development
1. Ask further conversion questions.
2. Practise rounding. Use number lines to decide how to round sports results to 1 d.p., to the nearest whole number … For example:
   If all results were rounded to the nearest whole number, which results would look the same?
   What if you rounded to 1 d.p.?
3. Consolidate written methods of addition and subtraction using questions as above.
Hundreds and thousands

0.056 × 0.0072 = 

5.6 × 72 = 

5600 × 0.072 = 

5.6 × 7.2 = 

56 × 0.072 = 

560 × 7.2 = 

0.56 × 0.72 = 

56000 × 720 = 

56 × 72 = 4032

From level 3 to level 5 in mathematics – part 2 | Course handbook | Section 3.2
<table>
<thead>
<tr>
<th>Javelin</th>
<th>Long jump</th>
<th>Pole vaulting</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong> 62.29 m</td>
<td><strong>A</strong> 7.185 m</td>
<td><strong>A</strong> 5.90 m</td>
</tr>
<tr>
<td><strong>B</strong> 62.305 m</td>
<td><strong>B</strong> 701.8 cm</td>
<td><strong>B</strong> 5.095 m</td>
</tr>
<tr>
<td><strong>C</strong> 62.3 m</td>
<td><strong>C</strong> 7049 mm</td>
<td><strong>C</strong> 500.9 cm</td>
</tr>
<tr>
<td><strong>D</strong> 62.35 m</td>
<td><strong>D</strong> 700.5 cm</td>
<td><strong>D</strong> 5090.5 mm</td>
</tr>
<tr>
<td><strong>E</strong> 62.285 m</td>
<td><strong>E</strong> 7.108 m</td>
<td><strong>E</strong> 0.005 945 km</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1st</th>
<th>1st</th>
<th>1st</th>
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<tbody>
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<td>2nd</td>
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<td>3rd</td>
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<td>4th</td>
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<tr>
<td>5th</td>
<td>5th</td>
<td>5th</td>
</tr>
</tbody>
</table>
Add-ons are part of the T5 (‘Targeting level 5’) pack. There are twelve Add-ons, each covering one of the major areas of mathematics drawn from the level 4–6 Key Stage 3 National Curriculum test papers, with a particular emphasis on those areas crucial for achieving level 5.

Each Add-on includes four to seven selected questions (mostly at level 4 and level 5) on the following topics:

1. Number: place value
2. Number: money problems
3. Number: percentages and fractions
4. Algebra: sequences and patterns
5. Algebra: expressions
6. Shape and space: area and perimeter
7. Shape and space: 2-D to 3-D
8. Shape and space: shapes and angles
9. Shape and space: transformations
10. Handling data: interpreting data
11. Handling data: pie charts
12. Handling data: probability

Using Add-ons

Add-ons can be used in a variety of ways.

- Add-ons can be programmed into normal lessons as the content for the middle part of the lesson. Most will take less than 20 minutes to complete. It is important that pupils check their work, and 5–10 minutes of the lesson could be used to go over the work that has been completed. Pupils could work individually or in pairs; it may be helpful to ask individual pupils to explain their working to the rest of the class or to a smaller group. At all times pupils need to have feedback on the work that they have done.

- Add-ons are suitable for homework assignments, but again it is important that the work is completed and that there is formative feedback.

- Add-ons can be used as an assessment tool. The content covers the majority of the level 5 (Years 7–8) key learning objectives. Successful completion of the questions of a particular Add-on suggests that pupils have a sound knowledge of that key learning objective.
1 Cards

Here are some number cards.

0 1 2 3 4 5

Joan picked these three cards.

She made the number 314 with her cards.

(a) Make a smaller number with Joan’s three cards.

(b) Make the biggest number you can with Joan’s three cards.

(c) Joan made the number 314 with her three cards. Which extra card should she pick to make her number 10 times as big?

What number is 10 times as big as 314?

(d) Andy has these cards.

0 1 2 3 4 5 •

He made the number 42.5 with four of his cards. Use some of Andy’s cards to show the number 10 times as big as 42.5

Use some of Andy’s cards to show the number 100 times as big as 42.5
2 Digits

(a) Claire puts a 2-digit whole number into her calculator. She multiplies the number by 10. Fill in one other digit which you know must be on the calculator display.

(b) Claire starts again with the same 2-digit whole number. This time she multiplies it by 100. Fill in all the digits that might be on the calculator display.

3 Missing numbers

Write one number at the end of each equation to make it correct.

Example \(26 + 34 = 16 + 44\)

(a) \(38 + 17 = 28 + \quad \) 1 mark

(b) \(38 - 17 = 28 - \quad \) 1 mark

(c) \(40 \times 10 = 4 \times \quad \) 1 mark

(d) \(7000 \div 100 = 700 \div \quad \) 1 mark
4 Arrangements

Here are some number cards.

1 7 3 5

You can use each card once to make the number 1735, like this.

1 7 3 5

Use some of the four number cards to make numbers that are as close as possible to the numbers written below.

Example

8000 → 7 5 3 1

You must not use the same card more than once in each answer.

4000 → 1 mark

1500 → 1 mark

1600 → 1 mark
5 Numbers

Here is a list of numbers.

-7 -5 -3 -1 0 2 4 6

You can choose some of the numbers from the list and add them to find their total.

For example

6 + 1 = 5

(a) Choose two of the numbers from the list which have a total of 3

_______ + _______ = 3

1 mark

(b) Choose two of the numbers from the list which have a total of -1

_______ + _______ = -1

1 mark

(c) What is the total of all eight of the numbers on the list?

_______

1 mark

(d) Choose the three numbers from the list which have the lowest possible total.

Write the three numbers and their total.

You must not use the same number more than once.

_______ + _______ + _______ = _______

2 marks
Stingers are Key Stage 3 National Curriculum test questions that are used to stimulate whole-class discussion and promote problem solving. They let both teacher and pupils focus on a learning objective and assess pupils’ knowledge and understanding. The material is particularly relevant to pupils targeting level 5.

Stingers provide plenary discussion for lessons. They can be run through a computer and projector. Alternatively they can be copied onto an OHT. The content would be difficult to write on a board, but could be photocopied for small-group discussion.

It is important to allow discussion by members of the class and wherever possible allow pupils the opportunity to explain their reasoning to the whole class.

Content

1. Decimal numbers and measures  Problem solving
2. Weigh it up  Problem solving
3. Car parking  Problem solving
4. Travel passes  Money problem
5. Shortcuts with percentages  Percentages
6. Adding, ordering and equivalents  Fractions
7. Ages  Ratio
8. Huts  Sequences
9. Substitution  Substitution
10. Simplifying and solving  Equations and expressions
11. Folding rectangles  Area and perimeter
12. It’s in the net  Nets and solids
13. Star pattern  Angles and symmetry
14. Favourite books  Pie charts
15. Fair games  Probability
16. Game scores  Averages

The teaching notes on the CD-ROM suggest activities to accompany most of the Stingers.
Decimal numbers and measures

1 Some children are comparing their heights.
   
   (a) Peter’s height is 0.9 m.
       Lucy is 0.3 m taller than Peter.
       What is Lucy’s height?

   (b) Lee’s height is 1.45 m.
       Misha is 0.3 m shorter.
       What is Misha’s height?

   (c) Zita’s height is 1.7 m.
       What is Zita’s height in centimetres?

(2002, 4–6, P1, Q7)

2 A scale measures in grams and ounces.

   (a) About how many ounces is 400 grams?

   (b) About how many grams is 8 ounces?

   (c) About how many ounces is 1 kilogram?

(2002, 4–6, P2, Q7)
Weigh it up

There are two small tins and one big tin on these scales.

The two small tins each have the same mass. The mass of the big tin is 2.6 kg.

What is the mass of one small tin?
Show your working.

(2003, 4–6, P1, Q7)
The Top Ten materials comprise 15 short tests to practise mental mathematics and support the needs of pupils for the higher level mental mathematics paper (papers A and B, which are taken by those pupils entering at level 4–6, level 5–7 and level 6–8).

There are three sets of tests (X, Y and Z), each providing practice for different topic areas in the mental mathematics tests. There are five tests for each set, with 10 questions in each test. The same number question in each set of tests is on the same topic area. For example, in sets X1 to X5 question 1 is on place value, involving multiplying and dividing by 10, 100 and 1000.

The decision to have these 30 question areas was based on the frequency and types of question that occur in the Key Stage 3 National Curriculum test mental mathematics papers.

All of the questions used are taken from Key Stage 3 mental mathematics papers A and B for the years 1997–2001, allowing the use of 2002, 2003 and 2004 as practice tests in the classroom.

Using the Top Ten tests

With 15 tests it is important to use these in an efficient and well-paced manner and at the same time allow pupils to develop their knowledge and understanding.

- One way of using the tests is to use the last 20 minutes of two mathematics lessons a week (for the spring term), using the first 10 minutes to review two topic areas that occur on the paper and the last 10 minutes for the test and to mark and record results.
- Sitting the test and marking it should only take 10 minutes.
- Pupils should record results on the pupil record sheet. Use of the pupil record is an essential part of Top Ten.
- Before and/or after the test one or two areas within the test should be developed in the classroom to help raise achievement in those areas.
- Pupils should be encouraged to look at the questions they continue finding difficult and discuss the work with their peers or the teacher.
- It is possible to produce a 20-page booklet which includes the 15 tests, the pupil record sheet and other background information. This will fill five A4 folded sheets and will provide a record for pupils’ reflection. The 15 answer papers have been provided on the CD-ROM without the questions to allow a more ‘mental test’ approach, if you prefer it.

Top Ten provides a tightly focused approach to review pupils’ strengths and weaknesses and to focus effective teaching. It is expected that, through teaching and regular practice of the same areas of mathematics, pupils who find specific areas difficult will be able to understand and learn how to solve problems in these areas. Pupils who reflect on their own strengths and weaknesses will make more progress.
<table>
<thead>
<tr>
<th>Question</th>
<th>Support</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Multiply 6.08 by one thousand.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 Write one quarter as a decimal.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 35% of a number is 42. What is 70% of the number?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 One ticket costs £17.50. What will four tickets cost?</td>
<td></td>
<td>£17.50</td>
</tr>
<tr>
<td>5 Write an approximate answer to the calculation on your answer sheet.</td>
<td></td>
<td>198.7 ÷ 4.98</td>
</tr>
<tr>
<td>6 How many millilitres are in a litre?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 What is the area of the triangle on your answer sheet?</td>
<td></td>
<td>![10 cm 10 cm]</td>
</tr>
<tr>
<td>8 Simplify the expression on your answer sheet as fully as possible.</td>
<td></td>
<td>$y \times y \times y$</td>
</tr>
<tr>
<td>9 Look at the equation on your answer sheet. Work out the value of $5k$.</td>
<td></td>
<td>$3k = 18$</td>
</tr>
<tr>
<td>10 What is the mean of the four numbers on your answer sheet?</td>
<td></td>
<td>19, 21, 23, 37</td>
</tr>
</tbody>
</table>
### Section 3.6

**Using T5 materials in the spring term**

<table>
<thead>
<tr>
<th>Link to the Y9 intervention plan</th>
<th>Learning Objectives Selected from the Y9 Intervention Plan</th>
<th>Activities and Resources</th>
</tr>
</thead>
</table>
| Shape, space and measures 2      | • Use units of measurement to estimate, calculate and solve problems in everyday contexts involving length, area, volume, capacity, mass, time and angle.  
• Deduce and use formulae for the area of a triangle, parallelogram and trapezium;  
• Calculate areas of compound shapes made from rectangles and triangles.  
• Know and use the formula for the volume of a cuboid. | Snapper 9: Halving rectangles (area and perimeter)  
Add-on 6: Shape and space: area and perimeter  
Stinger 11: Folding rectangles (area and perimeter)  
Top Ten X2 and X3 |
| Number 3                         | • Recall known facts, including fraction to decimal conversions; use known facts to derive unknown facts.  
• Add and subtract integers and decimals with up to two places.  
• Multiply and divide integers and decimals, including by decimals such as 0.6 or 0.06; understand where to position the decimal point.  
• Calculate fractions of quantities. | Snapper 1: Hundreds and thousands (place value)  
Snapper 2: Sports results (place value)  
Add-on 1: Number: place value  
Stinger 1: Decimal numbers and measures (problem solving)  
Stinger 2: Weigh it up (problem solving)  
Stinger 3: Car parking (problem solving)  
Top Ten X1 and X4 |
| Algebra 3                        | • Consolidate understanding of the relationship between ratio and proportion.  
• Divide a quantity into two or more parts in a given ratio; use the unitary method to solve simple word problems involving ratio.  
• Solve simple problems. | Snapper 5: Year 9 maths (ratio)  
Add-on 2: Number: money problems  
Stinger 4: Travel passes (money problem)  
Stinger 7: Ages (ratio)  
Top Ten X5 and Y1 |
| Number 4                         | • Find and justify theoretical probabilities based on equally likely outcomes.  
• Know that if the probability of an event occurring is \( p \), then the probability of it not occurring is \( 1 - p \); find and record all possible mutually exclusive outcomes for single events. | Snapper 15: Fairground games (probability)  
Add-on 12: Handling data: probability  
Stinger 15: Fair games (probability)  
Top Ten Z3 and Z4 |
| Handling data 2                  |                                                            |                          |
### LEARNING OBJECTIVES SELECTED FROM THE Y9 INTERVENTION PLAN

<table>
<thead>
<tr>
<th>Link to the Y9 intervention plan</th>
<th>Activities and Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Shape, space and measures 3</strong></td>
<td>Snapper 10: Nets of cuboids (nets and solids)</td>
</tr>
<tr>
<td></td>
<td>Snapper 11: Angles and transformations (angles and symmetry)</td>
</tr>
<tr>
<td></td>
<td>Snapper 12: Transformations (transformations)</td>
</tr>
<tr>
<td></td>
<td>Add-on 7: Shape and space: 2-D to 3-D</td>
</tr>
<tr>
<td></td>
<td>Add-on 9: Shape and space: transformations</td>
</tr>
<tr>
<td></td>
<td>Stinger 12: It's in the net (nets and solids)</td>
</tr>
<tr>
<td></td>
<td>Top Ten Y4 and Y5</td>
</tr>
<tr>
<td><strong>Algebra 4</strong></td>
<td>Snapper 6: Arrows (sequences)</td>
</tr>
<tr>
<td><strong>Algebra 1 and 2 review</strong></td>
<td>Snapper 7: Twelve days of Christmas (expressions and equations)</td>
</tr>
<tr>
<td></td>
<td>Snapper 8: Substitution spider (substitution)</td>
</tr>
<tr>
<td></td>
<td>Add-on 4: Algebra: sequences and patterns</td>
</tr>
<tr>
<td></td>
<td>Add-on 5: Algebra: expressions</td>
</tr>
<tr>
<td></td>
<td>Stinger 8: Huts (sequences)</td>
</tr>
<tr>
<td></td>
<td>Stinger 9: Substitution (substitution)</td>
</tr>
<tr>
<td></td>
<td>Stinger 10: Simplifying and solving (equations and expressions)</td>
</tr>
<tr>
<td></td>
<td>Top Ten Z1, Z2 and Z5</td>
</tr>
<tr>
<td><strong>Solving problems and revision</strong></td>
<td>Snapper 3: Stepping stones to percentages (percentages)</td>
</tr>
<tr>
<td></td>
<td>Snapper 4: Stepping stones to fractions (fractions)</td>
</tr>
<tr>
<td></td>
<td>Snapper 13: Potato bar chart (interpreting data)</td>
</tr>
<tr>
<td></td>
<td>Snapper 14: Potato pie chart (pie charts)</td>
</tr>
<tr>
<td></td>
<td>Snapper 16: Mean maths (averages)</td>
</tr>
<tr>
<td></td>
<td>Add-on 3: Number: percentages and fractions</td>
</tr>
<tr>
<td></td>
<td>Add-on 8: Shape and space: shapes and angles</td>
</tr>
<tr>
<td></td>
<td>Add-on 10: Handling data: interpreting data</td>
</tr>
<tr>
<td></td>
<td>Add-on 11: Handling data: pie charts</td>
</tr>
<tr>
<td></td>
<td>Stinger 5: Shortcuts with percentages (percentages)</td>
</tr>
<tr>
<td></td>
<td>Stinger 6: Adding, ordering and equivalents (fractions)</td>
</tr>
<tr>
<td></td>
<td>Stinger 13: Star pattern (angles and symmetry)</td>
</tr>
<tr>
<td></td>
<td>Stinger 14: Favourite books (pie charts)</td>
</tr>
<tr>
<td></td>
<td>Stinger 16: Game scores (averages)</td>
</tr>
<tr>
<td></td>
<td>Top Ten Y2 and Y3; use mental tests from previous years as practice</td>
</tr>
</tbody>
</table>

### May

Key Stage 3 Tests
Learning from misconceptions in mathematics

Objectives

- To clarify differences between pupils’ mistakes, misunderstandings and misconceptions
- To discuss common misconceptions and their impact on pupils’ performance at level 5+
- To explore and discuss teaching strategies to counter misconceptions
- To model departmental discussions on misconceptions

Misconceptions from early experience

1. You can’t divide smaller numbers by larger ones
2. Division always makes numbers smaller
3. The more digits a number has, the larger is its value
4. Shapes with bigger areas have bigger perimeters
5. Letters represent particular numbers
6. ‘Equals’ means ‘makes’
Learning from misconceptions in mathematics

Areas of misconceptions

<table>
<thead>
<tr>
<th>Topic</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Fractions and decimals</td>
</tr>
<tr>
<td>B</td>
<td>Multiplication and division</td>
</tr>
<tr>
<td>C</td>
<td>Area and perimeter</td>
</tr>
<tr>
<td>D</td>
<td>Algebraic notation</td>
</tr>
</tbody>
</table>

Activities to counter misconceptions

- Collecting together different but equivalent representations of a concept or process (e.g. activities in topics A and B)
- Testing the validity of generalisations by asking whether they are always, sometimes or never true (e.g. activities in topics C and D)
Developing mental strategies

Key Stage 3
National Strategy
Developing mental strategies

Objectives

- To consider how to develop pupils’ mental calculation skills
- To develop teaching approaches that support the development of mental calculation

Understanding number and calculation strategies

- Repertoire of mental calculation strategies
- Understanding of numbers and the number system
- Reinforces and extends

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Developing mental strategies

Skills of mental calculation

1. Considering mental approaches as a first resort
2. Recalling number facts without hesitation
3. Using known facts to figure out new facts
4. Using a repertoire of strategies to work out mental calculations and being able to explain methods

Skills of mental calculation

5. Understanding and using relationships between the four operations to find answers and check results
6. Approximating calculations to judge whether or not an answer is reasonable
7. Solving problems

Guidance on teaching strategies for mental calculation

The booklet:
- lists number facts for rapid recall
- sets out expectations of calculation types
- lists strategies to help pupils calculate accurately and efficiently
- suggests suitable activities to help pupils develop and understand calculation methods
Developing mental strategies

Using the booklet: departments

As a department, use the booklet to:
- consider progression
- support planning to ensure mental calculation methods are taught
- explore strategies to enhance mental calculation
- check that schemes of work are pitched at the right level

Using the booklet: teachers

Teachers can use the booklet to:
- check the level of work
- consider a variety of strategies
- use a variety of activities to motivate and engage pupils
- backtrack when a pupil encounters a difficulty
- have as a constant reference and source of examples in lessons
3.0–3.2 Year 9 support

Key Stage 3 National Strategy
Year 9 support

Objectives

- To explore the materials available for a Year 9 intervention programme
- To consider the content for an intervention programme for pupils in Year 9 targeting level 5
- To consider how to integrate mathematical intervention materials into programmes of work

Approaches for pupils beginning Y7 working at L3 (or low L4)

- In Y7, use the sample medium-term plan: Y7 intervention
- In Y8, use the sample medium-term plan: Y8 intervention
- In Y9, use the sample medium-term plan: Y9 intervention
- This programme covers the curriculum needed to achieve L5 in Y9
Materials on the CD-ROM to support Year 9 pupils

- 10-4-10 (‘Ten-for-ten’) materials for holiday revision
- T5 (‘Targeting level 5’) materials

T5 materials

- Snappers: starter activities targeting particular teaching objectives
- Add-ons: Key Stage 3 questions targeting specific topics
- Stingers: plenary and whole-class discussion activities around key questions
- Top Ten: short tests supporting mental arithmetic practice
- Year 9 intervention plan for the spring term

Top Ten

- Three sets (X, Y, Z) of five tests, with 10 questions in each test
- Each test in a set practises the same ten areas of work – the same question number in each test is practice for the same objective
- Pupils mark each test and record their results on the grid provided
- Sitting, marking and recording the test should take only 10 minutes